PP-4

ANALYSIS AND DESIGN OF SLOW-WAVE STRUCTURES USING AN INTEGRAL EQUATION APPROACH

Thomas G. Livernois and Pisti B. Katehi

Electrical Engineering and Computer Science Department,
The University of Michigan, Ann Arbor, MI 48109-2122

ABSTRACT

An integral equation formulation which yields dispersion characteristics for planar transmission lines on layered, lossy substrates is presented. Galerkin's procedure in the space domain is used and roots of the resulting characteristic equation provide the desired phase and attenuation constants. Numerical results are compared to those found in the literature for the MIS slow-wave structure.

1. INTRODUCTION

MIS structures have been studied by several researchers and are used widely in related MIC's. The slowing effect can be applied to many devices such as: i) delay lines, ii) phase shifters, iii) tunable filters, and iv) others. The early work of Hasegawa et al [1], provided useful physical insight to the electromagnetic characteristics of the MIS microstrip transmission line, as do several subsequent articles, [2] - [7]. Unfortunately, some of this work is restricted to relatively low frequencies, thus, it is not useful for designing state-of-the-art monolithic MIC's. The remaining methods, namely, Finite-Element Analysis (FEA) and Spectral Domain Analysis (SDA), are somewhat cumbersome to work with. Consequently, accurate design criteria are difficult to obtain. This paper outlines a rigorous method for characterizing shielded, layered, planar transmission lines. The presented approach, even if based on an integral equation method, results in relatively simple design equations which can be programmed very efficiently in a personal computer. Dispersion characteristics found by this technique for the MIS microstrip transmission line are compared to published theoretical and experimental results.

II. THEORY

Every planar structure is characterized by a coupled set of equations which relate the Green's function for the metallic waveguide (Figure 1) to the microstrip currents carried by the perfectly conducting strip. This set can be put into the following form:

$$\begin{bmatrix} G_{yy} & G_{zy} \\ G_{yz} & G_{zz} \end{bmatrix} * \begin{bmatrix} J_y \\ J_z \end{bmatrix} = \begin{bmatrix} E_y \\ E_z \end{bmatrix} \tag{1}$$

In this approach, the four components of the matrix are expressed in terms of LSM and LSE waves generated by the appropriate infinitesimal electric currents, [8]. The expansions for J_y and J_z are chosen to satisfy their respective edge conditions. The primary advantage of this technique

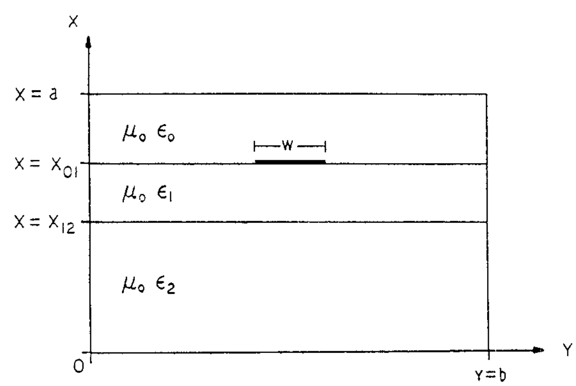

Fig. 1. Geometry of MIS slow-wave structure. (a = 1.5mm, b = 10.0mm, x_{01} = 0.251mm, x_{12} = 0.250mm)

results from the orthogonality of LSM and LSE modes. When solving for amplitude coefficients of the generating vector potentials, the boundary conditions on all tangential fields are invoked. This results in an apparent inhomogeneous 4x4 system of equations. However, this system decouples into two 2x2 sets of equations which relate LSM and LSE amplitude coefficients separately. The convolution integrals resulting from (1) are evaluated in closed form. Using one expansion term for J_y and J_z and applying the Galerkin's procedure to (1) shows:

$$\begin{bmatrix} \sum\limits_{\substack{m=1\\ odd}}^{M} P_{1m} & \sum\limits_{\substack{m=1\\ odd}}^{M} Q_{1m} \\ \sum\limits_{\substack{m=1\\ odd}}^{M} S_{1m} & \sum\limits_{\substack{m=1\\ odd}}^{M} U_{1m} \end{bmatrix} \begin{bmatrix} c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2)$$

where c_1 is the unknown amplitude coefficient for the first expansion term of J_y and d_1 similarly results from J_z. In equation (2) the expressions for P_{1m}, Q_{1m}, S_{1m} and U_{1m} are rather simple combinations of Bessel and Trigonometric functions, and are given in [9]. Setting the determinant of the current amplitude matrix to zero and solving for its roots yields the complex microstrip propagation constant $k_z^{ms} = \beta - j\alpha$.

III. NUMERICAL RESULTS

The dispersion characteristics given in this section are for the MIS structure with dimensions given in figure 1. Two different microstrip widths are considered. Region 2 is the lossy Si substrate with $\varepsilon_{r2} = 12$ and region 1 is the SiO_2 insulating region with $\varepsilon_{r1} = 4$. The effects of the induced conduction current are incorporated into a complex permittivity in region 2. The normalized wavelength and attenuation constant for different cases are plotted in figures 2 - 5. Good convergence was obtained using one expansion term for the microstrip current and M = 501 in the four truncated series in (2). Roots of the matrix were found using Mueller's method with deflation.

Good agreement between this theory, FEA, SDA, and experiment for λ/λ_o and α is found when W = 160μ. This data is shown in figures 2 and 3.

Results for the wider strip, given in figures 4 and 5 with W = 600μm, show discrepancies between this theory and the spectral domain approach for larger substrate conductivities. For the case σ = 1000 and f = 1 GHz the spectral analysis finds a very low normalized wavelength of about 0.04. This value is unacceptable considering that the Si substrate is five skin depths thick. As a result, the electromagnetic fields are virtually shielded from the semiconducting layer. This drives the line into the skin effect and not the slow wave mode. Results derived by the method presented in this paper indicate such a tendency. Curves generated from the parallel plate analysis (applicable to wide microstrip), [1], are also plotted in Figures 4 and 5 and are in agreement with our theoretical data. Figures 4, 5 also show experimental results plotted for various cases. Good quantitative agreement was found for smaller substrate conductivities while qualitative tendencies are observed for larger substrate conductivities.

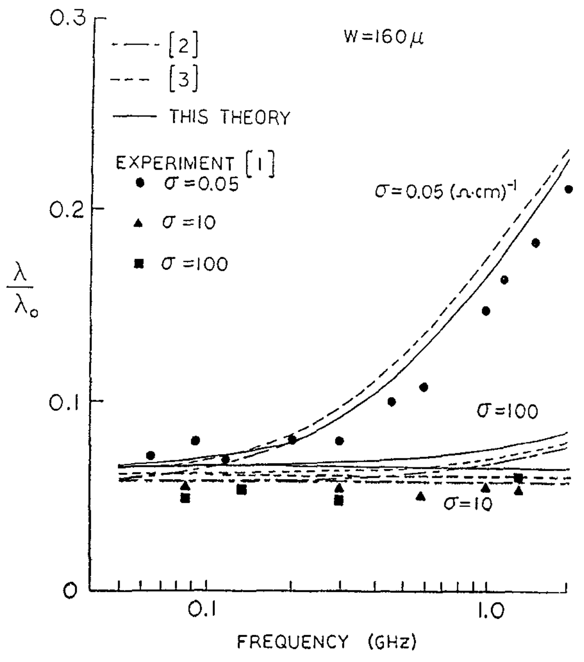

Fig. 2. Comparison of normalized wavelength with SDA [2], finite element [3], and experimental results [1], for W = 160μ

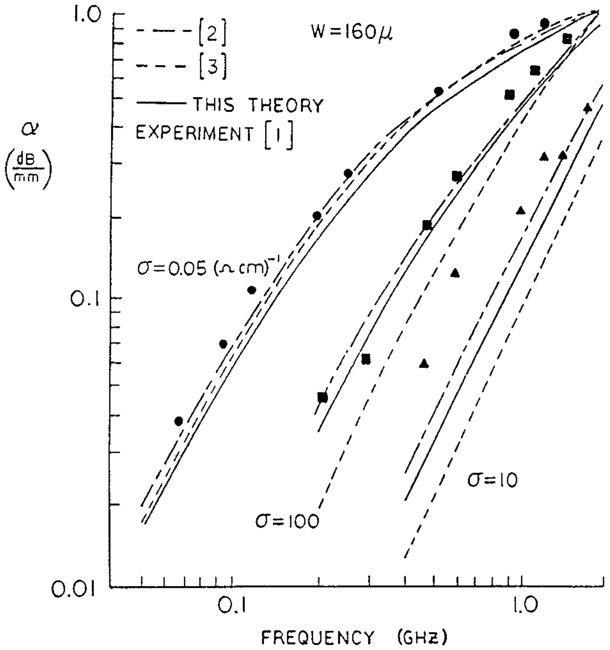

Fig. 3. Comparison of attenuation constant with SDA [2], finite element [3], and experimental results [1], for W = 160μ

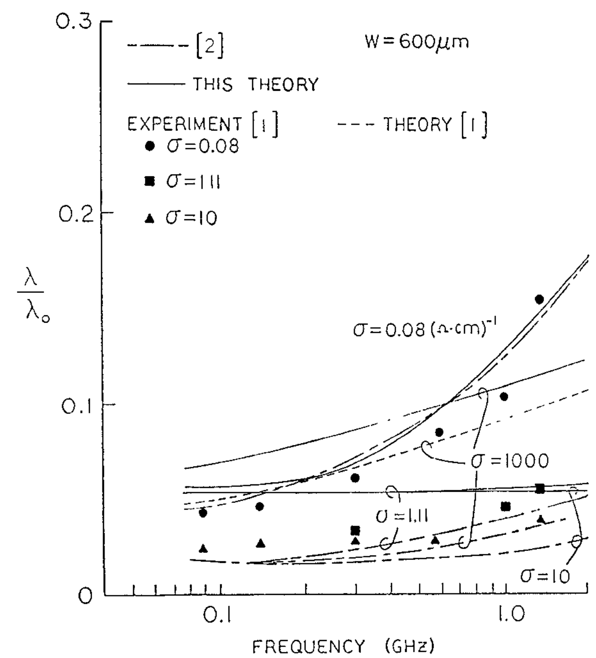

Fig. 4. Comparison of normalized wavelength with SDA [2] and experimental and parallel plate model results [1], for w = 600μ.

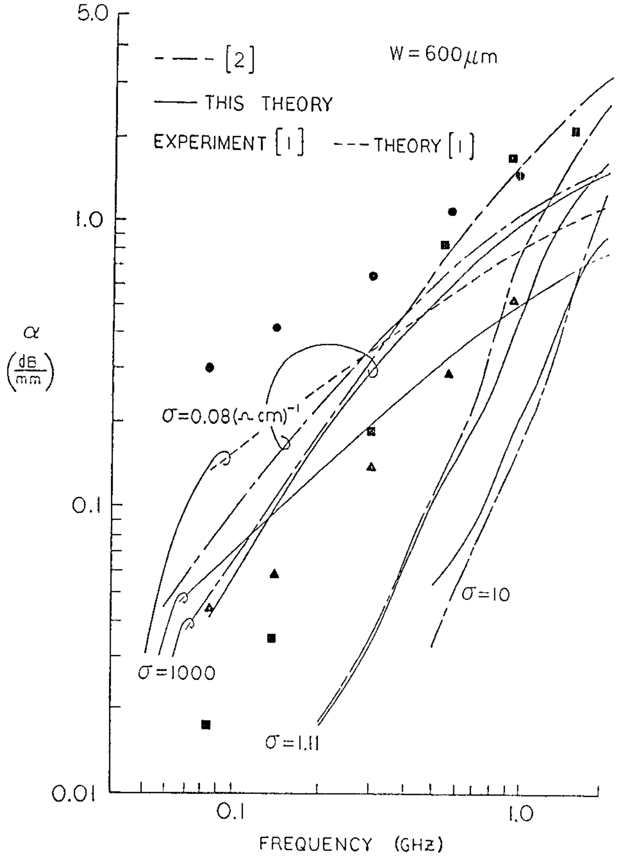

Fig. 5. Comparison of attenuation constant with SDA [2] and experimental and parallel plate model results [1], for w = 600μ

IV. CONCLUSION

An efficient, accurate method useful for characterizing layered, planar transmission lines has been presented in this paper. Numerical results were compared to other published work.

This method has been proved very accurate and efficient for studying lines on insulator-semiconductor substrates. The technique is based on an integral equation formulation and results in design equations which can be simply programmed on a personal computer. The validity of the numerical results was verified by comparing to available theoretical and experimental data.

V. ACKNOWLEDGEMENTS

This work has been sponsored by the National Science Foundation under the contract ECS-8602530.

REFERENCES

1. H. Hasegawa, M. Furukawa, H. Yanai: "Properties of Microstrip Line on Si-SiO_2 System" IEEE Trans. Microwave Theory and Techniques, vol. MTT-19 No. 11, November 1971, pp. 869-881.

2. P. Kennis and L. Faucon, "Rigorous Analysis of Planar MIS Transmission Lines," *Electron. Lett.*, vol. 17, no. 13, pp. 454-456, June 1981.

3. M. Aubourg, J. Villotte, F. Godon and Y. Garault, "Finite Element Analysis of Lossy Waveguides - Applications to Microstrip Lines on Semiconductor Substrate," IEEE Trans., MTT-31, No. 4, pp.326-330, April 1983.

4. D. Jager, "Slow Wave Propagation Along Variable Schottky Contact Line," IEEE Trans., *Microwave Theory Tech.*, vol. MTT-24, pp. 566-573, Sept. 1976.

5. Y.C. Shih and T. Itoh, "Analysis of Printed Transmission Lines for Monolithic Integrated Circuits," *Electron. Lett.*, vol. 18, No. 14, pp. 585-586, July 1982.

6. C. Tzuang and T. Itoh, "Finite-Element Analysis of Slow-Wave Schottky Contact Printed Lines," IEEE Trans., MTT-34, No. 12, pp. 1483-1489, December 1986.

7. C. Krowne, "Slow-Wave Propagation in Generalized Cylindrical Waveguides Loaded with a Semiconductor," *Int. J. Electronics.*, vol. 58, No. 2, pp. 249-269, 1985.

8. R. Harrington, "Time Harmonic Electromagnetic Fields," New York: McGraw-Hill, 1961.

9. T.G. Livernois and P.B. Katehi, "A Generalized Method for Deriving the Space-Domain Green's Function in a Shielded, Multilayer Substrate Structure with Applications to MIS Slow-Wave Transmission Lines," Submitted to IEEE Trans. on Microwave Theory and Techniques.